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Superstring dualities, Dirichlet branes and the small scale structure of space

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Abstract

We give a broad overview of superstring duality, Dirichlet branes, and some implications of both for questions about the structure of space-time at short distances.

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1 Introduction

Superstring theory has been studied intensively since 1984, when the discovery by Green and Schwarz of anomaly cancellation convinced many physicists that it provides a consistent theory of perturbative quantum gravity, gauge interactions and chiral matter. The basic difficulties in quantizing general relativity and supergravity (non-renormalizability or at least strong coupling at the Planck scale) are visible at low order in the loop expansion, while superstring theory was shown to be well-defined and finite to all orders.

Although this was quite an achievement, and much has been understood at this level, it has been clear for some time that to answer the questions of direct physical interest, one needs non-perturbative results. These questions fall into two broad areas.

The first question is how does the standard model of particle physics emerge at low energies, those far below the natural scales of string theory and quantum gravity. All plausible scenarios for this so far involve supersymmetry at low energy, and superstring theory leads to reasonable supersymmetric extensions of the standard model given remarkably few assumptions, as explained in [15]. Supersymmetry must then be broken by non-perturbative effects, and detailed predictions depend crucially on this physics.

The second question is to understand situations involving strong gravitational effects. The most famous examples are the by now classic problems of black hole physics, such as what happens to a black hole at the end point of Hawking radiation, and in particular is this process governed by unitary quantum evolution. Many simpler situations exist as well. For example, consider a limit in which the volume of a non-contractible cycle in an internal space (e.g. a Calabi-Yau) goes to zero, and its curvature diverges. It has long been thought that our conventional ideas of space-time must be modified in such extreme situations and at very short distances.

Over the last two years, revolutionary progress in supersymmetric field theory and superstring theory has brought some of these questions within reach. Many of these developments were described by Dijkgraaf, Greene and other lecturers at this school. At the time of the school, the subject was evolving extremely rapidly, and the developments of the next few months greatly extended the reach of the new ideas and significantly clarified them. Thus, rather than artificially limit the discussion to the perspective of summer 1995, it appeared to me to be far more useful to adopt a later perspective

(summer 1996) while addressing some of the themes which came up in lectures and discussions at and inspired by the school.

The present contribution is a broad overview of superstring duality, Dirichlet branes, and the implications of both for questions about the structure of space-time at short distances. It is intended to be readable by physicists and mathematicians without a detailed knowledge of string theory, and serve as an introduction to more extensive reviews such as [31, 32] for superstring duality, [6, 27] for Dirichlet branes and [13] for M theory. (Another overview is [26].) Space did not permit mentioning many of the important developments; in particular we almost completely omit the topic of duality in compactified string theories. Many of the concepts were discovered in that context and thus we will not attempt to describe the history but again refer to these reviews. (A few of the papers which must be mentioned are [20, 29, 34, 39]).

After a brief discussion of field theoretic duality, we describe the dualities between the five ten-dimensional superstring theories and eleven-dimensional “M theory.” We then give an introduction to Dirichlet branes (D-branes), a particularly simple class of soliton in superstring theory, which enters into and simplifies many arguments in duality. We then give an overview of the works [9, 11, 22, 33], which showed why Dirichlet branes are particularly relevant in studying the nature of space-time at short distances, and combine this with a result of [12] – D-branes can see non-trivial topology in a novel way – to see that the topology and geometry relevant for general relativity must be embedded in a larger, essentially non-commutative structure in string theory. Finally, we discuss a few of the many open questions.

2 Duality and Solitons in Supersymmetric field theory

The central ideas behind the new developments are strong-weak coupling duality in abelian gauge theories, and precise bounds on the masses of “BPS states” in supersymmetric theories. Gauge theories and string theories have a rich spectrum of solitonic states, both particles and “ p -branes,” objects extended in p spatial dimensions. Classically, their masses and tensions typically behave as $m \sim 1/g^2$ in terms of a conventional three-point coupling g . In certain string theories (as we will discuss), one finds solitons with

$m \sim 1/g$. Some of these, the BPS states, belong to reduced multiplets of supersymmetry, and for them, the classical mass formula is valid quantum mechanically as well.² Thus, as g becomes large, the solitons become the lightest states and dominate the dynamics of the theory.

The simplest example of how this statement can be made precise is the “proof” of duality for four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory (henceforth, SYM). This theory has a moduli space of vacua and at generic points the gauge symmetry is broken to the Cartan subalgebra of the gauge group. The other generators of the group correspond to massive gauge bosons, with mass m set by the scale of gauge symmetry breaking, and these are the lightest charged states in the theory. If we work at energies far below m , the theory is effectively the same as abelian gauge theory, a non-interacting theory. (All of these points will reappear in our discussion of D-branes in section 4, with a rather different physical picture.)

The BPS solitons are magnetic monopoles with mass m/g^2 , and for $g > 1$, they become the lightest charged states. After a duality transformation exchanging electric and magnetic charge, we get a weakly coupled gauge theory with these charged particles. Now – and this is the main point where we use the fact that this is $\mathcal{N} = 4$ SYM – the monopoles form supersymmetry multiplets containing spin 1 particles which are identical to the original gauge boson multiplets. We only know of one sensible field theory containing charged spin 1 particles, gauge theory. Thus, even without having a precise change of variables from the original Yang-Mills theory to the dual theory, we can assert that for sufficiently large g , the dual theory must be a gauge theory. Finally, $\mathcal{N} = 4$ SYM is the unique gauge theory with these degrees of freedom and symmetry. The final result, a dual theory isomorphic to the original theory at weak coupling, is special to this example, but the line of reasoning is general.

All this was Montonen and Olive’s original argument for their conjecture of duality [24] and it is obviously far from a mathematical or even physicist’s proof. However many physicists are convinced by the argument, not just by the beauty of the result and the large framework it has been fit into, but

² This will be true in the cases we discuss, which have enough supersymmetry to protect the low energy effective Lagrangian from quantum corrections. The general statement is that the classical mass formula derived from the exact low energy effective Lagrangian is valid. This is an equally powerful statement which (for example) was central in Seiberg and Witten’s solution of $\mathcal{N} = 2$ super Yang-Mills theory.

because upon reflection, it is hard to come up with a way for it to fail which would not be even more surprising than the duality conjecture.

The first step in a proof would be to show that $\mathcal{N} = 4$ SYM actually exists (there is no ‘supersymmetry anomaly’). It is unlikely that there is any regulated form of the theory with the supersymmetry (besides string theory!) and nobody knows how to even begin to prove this, but in every known anomalous physical theory (one in which a classical symmetry is not present in the quantum theory), there is a perturbative or semiclassical computation which exhibits the anomaly. These computations have been done for $\mathcal{N} = 4$ SYM and no anomaly is seen.

One might imagine that some non-BPS states of which we are unaware are the true lightest states. They would have to be uncharged (the BPS mass is a lower bound for charged states) and this forces their couplings to the abelian gauge fields to be non-renormalizable and (for generic parameters) less important at low energies than the minimal gauge theory couplings. Although these points speak against this possibility, it has not been shown to be inconsistent, and this is perhaps the most reasonable way the duality hypothesis could fail. However, $\mathcal{N} = 4$ supersymmetry is a strong constraint and no sensible scenario of this sort has been proposed.

Then, assuming the conventional principles of field theory apply at low energies (since the theory is essentially Maxwell’s theory in this regime, this is very well motivated), the remaining element of the argument which is not straightforward to check is the assertion that the only sensible field theories containing particles with spin 1 (and no higher) are gauge theories. Although this is not easy to prove, it has been a central point in theoretical physics for some time (it is the foundation of the theoretical justification of the Standard Model) and almost all physicists are convinced of it.

This physical background may help explain why, once the idea of duality was taken seriously, the remarkable paradigm shift of 1994-1995 was accepted so quickly. We can add to this the impact of Seiberg’s work on supersymmetric gauge theory and especially the celebrated Seiberg-Witten solution of $\mathcal{N} = 2$ SYM, which after twenty years of effort gave the first analytic results on confinement in four-dimensional gauge theory, and demonstrated that these ideas could solve problems of universally accepted physical importance [21].

3 Duality and Solitons in Superstring theory

We begin by reviewing the field content and low energy effective Lagrangians of superstring theories in ten dimensions. The particle content is found by quantizing a single superstring, and each massless particle is associated with a field in the effective Lagrangian. This analysis is described in many references such as [15]. Although it is too long a story to repeat here, we outline it before quoting the Lagrangians, to make the point that their field content, which at first sight may seem somewhat arbitrary, has a simple origin whose essence can be summarized fairly briefly. Given these fields, it has been shown that in each case there is a unique low energy Lagrangian [36].

3.1 Spectrum of superstring theories

The motion of a superstring world-sheet is governed by a two-dimensional quantum field theory, whose degrees of freedom always include a map embedding the world-sheet into space-time. A specific theory is characterized by the additional degrees of freedom which appear. In the type IIa and IIb closed string theories, these are two ten-dimensional spinors θ^1 and θ^2 , giving rise to $\mathcal{N} = 2$ supersymmetry. The two cases are distinguished by their relative chiralities – opposite for IIa (so the theory has a parity symmetry) and the same for IIb. The heterotic strings are closed strings with $\mathcal{N} = 1$ supersymmetry and thus a single ten-dimensional right moving spinor θ^1 , and 32 left moving fermions λ^I . These are singlets under Lorentz symmetry but admit an obvious action of $SO(32)$. The full definition of the theory specifies a sum over sectors with various twisted boundary conditions, distinguishing theories with $SO(32)$ or $E_8 \times E_8$ gauge symmetry.

The type I string contains open strings, and a subset of the type IIb closed strings as well. The boundary conditions of an open string relate θ^1 to θ^2 and the result is that the pair behaves as a single ten-dimensional spinor degree of freedom, and the theory has only $\mathcal{N} = 1$ supersymmetry. Furthermore, the existence of the boundaries allows us to introduce a discrete degree of freedom or “Chan-Paton factor” at each boundary. Open string states are labelled by a pair of such choices, say $1 \leq I, J \leq N$. We could notate a field creating an open string with a specified embedding $X(\sigma)$ and auxiliary degrees of freedom as $\phi_J^I[X(\sigma), \theta(\sigma)]$.

The basic open string interaction is a joining of strings at the endpoints,

and if we make the natural definition that the Chan-Paton factors must agree, it will include a matrix multiplication $\sum_J \phi_J^I \phi_K^J$. Thus open strings come with a natural algebraic structure, from which arises the Lie algebra structure required for non-abelian gauge symmetry.³ At the quantum level, consistency requires the gauge group to be $SO(32)$. This comes from the choice $N = 32$ and the identification $\phi_J^I[X(\sigma), \theta(\sigma)] = \phi_I^J[X(-\sigma), \theta(-\sigma)]$, which expresses the identification of the open string with its orientation reversal, and makes the vector potential antisymmetric. These conditions are forced on us only in the particular case of open strings in ten dimensions.

The essential step in finding the massless spectrum is to identify the zero modes of the additional world-sheet degrees of freedom, as the massless one particle states must form a representation of their algebra. The result for the Lorentz non-singlet fermionic zero modes (those carrying spin) is easily stated – there is one for each supercharge which acts non-trivially on the state. More explicitly, a massless particle state admits the little group symmetry $SO(8) \subset SO(9, 1)$, and each ten-dimensional world-sheet spinor contributes a zero mode $p_\mu \gamma^\mu \theta = 0$, a spinor of $SO(8)$. Each has eight real components and thus the massless states of the type II strings admit an action of a Clifford algebra with 16 generators and come in a 256 component multiplet of supersymmetry, while the massless states of the type I and heterotic strings admit an action of a Clifford algebra with 8 generators and come in multiplets with at least 16 components. The Lorentz representations of the states are thus determined and it is a short exercise to show both that a graviton is present and – the non-trivial result we will use from this analysis – that the type II multiplets contain an two-form gauge potential and additional bosonic states in the bispinor of $SO(8)$. The bispinor is equivalent to the direct sum of antisymmetric forms which appear in the effective Lagrangians.

³The seminar actually given at Les Houches involved a lengthy discussion of extensions of this algebraic structure to the entire space of loop functionals, provided by Witten's string field theory [42] and by matrix models [14]. It is a mark of how deeply the recent developments cut that the fundamental role of such constructions has become completely unclear. The strongest argument against their fundamental role is the existence of limits of the theory which do not contain fundamental strings.

3.2 Effective Lagrangians for superstring theories

The type II strings have $\mathcal{N} = 2$, $d = 10$ supersymmetry and thus a metric $g_{\mu\nu}$, two ten-dimensional gravitinos, and additional spinors to fill out 128 fermionic states. The additional bosonic fields are the scalar ‘dilaton’ ϕ , a differential 2-form gauge potential $B^{(2)}$, and a sum of odd rank (for IIa) or even rank (for IIb) $p + 1$ -form gauge potentials $C^{(p+1)}$.⁴ The bosonic part of the type II effective action is then⁵

$$\mathcal{L}_{II} = \int d^{10}x e^{-2\phi} \left(\sqrt{g}R + 4||d\phi||^2 + ||dB^{(2)}||^2 \right) + \sum_{\substack{p=0,2 \text{ IIa} \\ p=-1,1,3 \text{ IIb}}} ||dC^{(p+1)}||^2. \quad (1)$$

The heterotic theory contains a subset of these fields as well as non-abelian gauge fields with field strength F . Its bosonic effective action is

$$\mathcal{L}_{het} = \int d^{10}x e^{-2\phi} \left(\sqrt{g}R + 4||d\phi||^2 + ||dB^{(2)} - \omega_3||^2 + \frac{1}{4}\text{Tr}||F||^2 \right) \quad (2)$$

where ω_3 is a sum of Chern-Simons three-forms constructed from the spin (Lorentz) connection and the non-abelian gauge connection [15]. The type I theory contains a subset of the type IIb fields, and non-abelian gauge fields F from the open strings:

$$\mathcal{L}_I = \int d^{10}x e^{-2\phi} \left(\sqrt{g}R + 4||d\phi||^2 \right) + e^{-\phi} \text{Tr}||F||^2 + ||dC^{(2)} - \omega_3||^2. \quad (3)$$

The type II effective Lagrangians have gauge symmetries $\delta B^{(2)} = d\Lambda^{(1)}$ and $\delta C^{(p+1)} = d\lambda^{(p)}$. The standard gauge transformation laws are modified in the heterotic and type I theories, as is clear by the presence of the term ω_3 . The full story is a bit subtle and involves a cancellation of gauge anomalies after quantization (the Green-Schwarz mechanism [15]).

We did not write an overall coupling constant $1/\hbar$ or $1/G_N$ in front of the Lagrangians, because it can be absorbed by a shift of the dilaton ϕ and field redefinitions. However, there is a moduli space of vacua characterized

⁴ We denote the rank of a potential as $p + 1$ for reasons to be explained shortly. $B^{(2)}$ is often called the “Neveu-Schwarz” or NS two-form, and the $C^{(p+1)}$ are called “Ramond-Ramond” or RR forms, from their origins in the superstring spectrum.

⁵ The actions (1) and (6) also contain cubic Chern-Simons couplings, schematically $\int B \wedge dC \wedge dC$, but they are not needed to verify the statements made here.

by the expectation value $\langle\phi\rangle$ which will control an effective coupling constant $g_s = e^{\langle\phi\rangle}$. The presence of this scalar also raises an important point: there is a family of metrics (with parameter a)

$$G_{\mu\nu}^{(a)} \equiv e^{a\phi} g_{\mu\nu} \quad (4)$$

several of which naturally appear in the discussion. Lengths and masses must be quoted with respect to a particular metric or ‘frame.’ The only one with an obvious preferred status in the low energy Lagrangian is the Einstein metric, the one which is governed by the Einstein action $\int d^{10}x \sqrt{G} R[G]$. However, this is not the natural metric when a preferred (weakly coupled) BPS state is present, as the Einstein mass of the preferred state will usually depend on the coupling constant. The Lagrangians above are written using the “string metric,” in terms of which the tension T of a fundamental string is independent of the coupling. We implicitly set it to 1 above; when we want to make it explicit we use the standard constant $\alpha' \equiv 1/T$ of dimensions (length)² instead.

To find the strong coupling limit of the theories, we begin by classifying BPS solitons. The result is easy to state:

Claim 1 *For every $p + 1$ -form gauge potential, there is a unique associated electrically charged BPS p -brane, and a unique associated magnetically charged BPS $(6 - p)$ -brane.*

The associated topological quantities are just the electric and magnetic charges $Q_E = \int *dC$ and $Q_M = \int dC$ respectively. Thus we must explain what a p -brane is before proceeding.

3.3 p -branes

A p -brane is a solution of the classical equations of motion, independent of $p + 1$ coordinates y^μ and localized in the remaining (here $d = 9 - p$) coordinates, say around $x^i = 0$. We write the solution schematically as $\phi(x)$. Such solutions are often determined by the solution in a subsector with a topologically non-trivial field configuration. For example, a self-dual Yang-Mills configuration gives rise to a solution of Einstein-Yang-Mills (still depending on four coordinates), with metric non-trivial but determined by the gauge field. All BPS solitons have this topological character.

Solitonic solutions generally have continuous parameters (moduli or collective coordinates). Any solution which breaks translation invariance in d dimensions will have moduli $\phi(x; X) = \phi(x + X; 0)$. Others might be predicted by symmetry or appear non-generically – for example, the instanton has a scale size and an embedding in the gauge group.

Configurations with slowly varying moduli are approximate solutions. Choosing local coordinates m on the moduli space such that $\phi(x; m)$ is the solution with moduli m , the nearby configuration $\phi(x; m(y))$ will have action proportional to $\int d^{p+1}y (\partial m)^2$, so the p -brane admits fluctuations with arbitrarily small action.

Now we can regard the choice of $p+1$ -dimensional hyperplane (or “world-volume”) and map $m(y)$ from the world-volume to moduli space as specifying the state of the p -brane. The $m(y)$ behave as local fields on the world-volume, and a p -brane in isolation will be governed by an effective action derived by substituting the ansatz $\phi(x; m(y, t))$ into the original ten-dimensional action. This action will include the metric on moduli space, its supersymmetrization, couplings to other bulk fields, and so forth. Its low energy limit is largely determined by the symmetries broken by the soliton.

The universal part of the effective action governs the transverse fluctuations of the p -brane. It is determined by the charge Q_p and the original Poincaré invariance to be the Nambu action with a minimal gauge coupling,

$$S = \int T_p f^*(V) + Q_p f^*(C^{(p+1)}). \quad (5)$$

The tension T_p is proportional to Q_p with ratio a function of the vacuum moduli (here $\langle \phi \rangle$ or g_s) fixed by the BPS condition in the ten-dimensional field theory. $f : \mathbf{R}^{p+1} \rightarrow \mathbf{R}^{10}$ is the embedding, $f^*(V)$ is the volume element induced on the brane from the space-time metric $G_{\mu\nu}$, and $f^*(C^{(p+1)})$ is the pullback of the gauge potential. The other zero modes will have kinetic terms and interactions consistent with symmetry but dependent on the particular case.

To return to the example of the self-dual gauge field, it can be promoted to a solution of ten dimensional superstring theory which is localized in four and extended in six dimensions. This is a five-brane, an extended object filling five dimensions of space as well as time. Its non-zero action $\int \text{tr} F^2 = 4\pi^2$ and the coupling dependence in the Lagrangian determines its tension to be $T_{5h} = 4\pi^2/g_s^2$ in the heterotic string, and $T_{5I} = 4\pi^2/g_s$ in the type I string.

For $p = 1$, (5) is a form of the superstring action we might have used to begin a detailed discussion of superstring quantization. Thus a string soliton is identical to a fundamental string at this level. They differ in their additional interactions. A fundamental string (by definition) admits a limit (zero string coupling) in which all embeddings of its world-volume are possible, with precisely the action (5). For the string soliton (5) is an approximation which will break down (for example) when the world-sheet self-intersects. The basic hypothesis underlying superstring duality is that, despite this difference in our explicit definitions of the objects, they are continuously connected by varying the vacuum moduli. In a limit in which T_1 for a particular soliton string becomes much smaller than other dimensionful quantities in the theory, that string will become a fundamental string. This hypothesis has the same character as the arguments of section 2 but is beyond explicit verification, even for a single string. Nevertheless it is amply justified by the results.

Having explained what a p -brane is, we return to describing the BPS p -branes of the various string theories. First, the fundamental string gives rise to a BPS state, electrically charged under $B^{(2)}$. Note that there must be an S^7 surrounding the object to have a non-zero charge and thus only a fundamental string infinitely extended in space will actually be a BPS state; states of the original quantized loops of string are not BPS. Note also that an infinitely extended fundamental string in the type I theory is not BPS. It is not even a stable state – it can break into open strings.

To prove the claim, we also need to find the magnetic $B^{(2)}$ (or NS) five-brane, as well as p -branes charged under the gauge fields $C^{(p+1)}$. The coupling involving ω_3 can be used to show that in the heterotic string, the NS five-brane is just the five-brane associated with self-dual gauge fields. An NS five-brane also exists in type II as a conventional soliton solution.

The p -branes associated with the RR gauge fields $C^{(p+1)}$ can be found as solitons, and are equivalent to the D-branes described in the next section. In type I theory the five-brane associated with self-dual gauge fields is one of these. They have some unusual characteristics following from the mixed dilaton dependence of the action – notably, their tension has the unusual coupling dependence $T_p \sim 1/g_s$.

We can now determine the object which governs the strong coupling limit of each theory, using “Hull’s criterion.” [19] We would like to find the lightest of the various p -branes as $g \rightarrow \infty$, but we cannot simply compare tensions T_p for different p as these quantities have different dimension. In our examples

this will not be a serious problem, but in compactification to lower dimensions many objects appear (branes wrapped around homology cycles) and it is less obvious which object dominates. Hull's criterion states that it will be the object of smallest $m = (T_p)^{1/(p+1)}$. Branes of high p are strongly disfavored by this criterion, and by now there is good evidence that they do not play the role of fundamental objects.

We proceed to consider the cases.

3.4 Type IIa

Type IIa superstring theory has a particle (zero-brane) with $m \sim 1/g_s\sqrt{\alpha'}$ and a membrane (two-brane) with $T_2 \sim 1/g_s(\alpha')^{3/2}$. The strong coupling limit is controlled by the zero-brane.

This fact was used by Witten [39] to argue that the strong coupling limit of type IIa superstring theory is in fact an eleven-dimensional theory whose low energy limit is eleven-dimensional supergravity, but with space-time the manifold $\mathbf{R}^{10} \times S^1$. Eleven-dimensional supergravity has a metric, gravitino and a single 3-form potential, and the bosonic Lagrangian

$$\mathcal{L}_{11} = \frac{1}{l_p^9} \int d^{11}x \sqrt{g}R + ||dC^{(3)}||^2. \quad (6)$$

The theory has no dimensionless coupling constant; the effective coupling constant is $l_p E$, energy measured in Planck units.

It was long known that S^1 (Kaluza-Klein) dimensional reduction of (6) followed by a metric substitution (4) produced the IIa low energy Lagrangian (1). The $g_{11,11}$ component of the metric (determining the radius R_{11} of the S^1) becomes the ten-dimensional coupling, leading to the relation $R_{11} = g_s^{1/3}\sqrt{\alpha'}$. The Kaluza-Klein gauge field $g_{11,\mu}$ becomes $C_\mu^{(1)}$, the vector potential coupling to the zero-brane. Thus the ten-dimensional zero-brane charge is reinterpreted as momentum in the eleventh dimension, P_{11} . This momentum is quantized, $P_{11} = n/R_{11}$, and the BPS argument guarantees that the ten-dimensional mass m of an eleven-dimensional massless state is exactly P_{11} . The vanishing $m = 1/g_s$ of the zero-brane mass in the strong coupling limit is interpreted as the limit $R_{11} \rightarrow \infty$.

Eleven-dimensional supergravity is neither finite nor renormalizable, so its connection with superstring theory provides the first convincing argument that it indeed exists as the low energy limit of a well-defined quantum theory.

This theory has been dubbed “M theory” and subsequent developments have shown that it is at least as fundamental to the overall picture as the superstring theories, and possibly more so. Little is known about its definition beyond the low energy limit; we return to this subject in the conclusions.

3.5 Type IIB

Type IIB superstring theory has a string with $T_1 \sim 1/g_s \alpha'$ and a three-brane with $T_3 \sim 1/g_s \alpha'^2$. The strong coupling limit is controlled by the string and must be another superstring theory with chiral $\mathcal{N} = 2$ supersymmetry. Thus it is the IIB superstring – this theory is self-dual. The appropriate transformation of the Lagrangian exchanges $B^{(2)}$ and $C^{(2)}$, and this requires both an inversion $e^\phi \rightarrow e^{-\phi}$ of the coupling constant, and a metric substitution (4). The latter can also be described intuitively as the rescaling of α' which when combined with $g \rightarrow 1/g$ exchanges the two string tensions, fundamental string $T_1 = \frac{1}{\alpha'}$ with solitonic string $T'_1 = \frac{1}{g_s \alpha'}$.

This theory is particularly interesting as it is in fact symmetric under general $SL(2, \mathbf{Z})$ transformations on the doublet $(B^{(2)}, C^{(2)})$ and on the complex coupling $\tau = ie^{-\phi} + C^{(0)}$. The duality described so far is the generator $S : \tau \rightarrow -1/\tau$ and is usually called “S-duality” for this reason. An orbit of $SL(2, \mathbf{Z})$ contains a spectrum of dyonic strings with all $(B^{(2)}, C^{(2)})$ charges (p, q) , p and q relatively prime. [30]

This symmetry has a pretty eleven-dimensional origin – to see it, we need to explain T-duality, which relates the two type II strings. This is only present after compactifying a dimension on a circle, so let us consider type IIA on $\mathbf{R}^9 \times S^1$. When the radius of the S^1 goes below the string scale, the lightest state in the theory will be a string wound about the S^1 , producing a particle with mass $m = 2\pi R/\alpha'$. In fact there is a spectrum of multiply wound strings with all masses $2\pi Rn/\alpha'$, and just as in our discussion of the strong coupling limit of IIA, the natural interpretation is that a *new* tenth dimension has appeared, of radius $R' = \alpha'/2\pi R$. The new theory is a type II superstring with the same fundamental string, but a detailed treatment shows that introducing the new dimension flips the chirality conditions and turns IIA into IIB. [31]

Thus, IIB in nine dimensions is T-dual to IIA in nine dimensions, and can be obtained by compactifying M theory on a $\mathbf{R}^9 \times T^2$ with both radii small. Then the $SL(2, \mathbf{Z})$ action on IIB is just large diffeomorphisms of the

T^2 [30, 1].

3.6 Heterotic

The heterotic string theories admit the same reasoning, and it leads to the conclusion that they are dual to theories of five-branes. This is true in a sense, but it turns out that the five-branes are not the fundamental degrees of freedom of the dual theories.

The reason that we should not immediately accept the argument which worked so well for type II strings is that in $\mathcal{N} = 1$, $d = 10$ theories, the fundamental states need not be BPS solitons, as shown by the example of the type I string.

A little experimentation shows that the S-duality transformation $\phi \rightarrow -\phi$ can be extended to a duality transformation on all the massless fields, mapping the Lagrangian (2) into (3). This leads to the conjecture that the $SO(32)$ heterotic string is dual to the type I string. The heterotic five-brane will then be dual to a solitonic object, the five-brane solution of the type I effective Lagrangian.

This duality will be easier to establish in the reverse direction, since the fundamental heterotic string is a BPS soliton.

3.7 Type I

The type I string theory has $C^{(2)}$, supporting a BPS string soliton and dual five-brane. A strong test of the conjectured duality is that this string must have the zero modes of the $SO(32)$ heterotic string. It is quite a task to show that the string soliton solution of the field theory indeed has these zero modes, and this is where the Dirichlet brane comes to the rescue, as we will describe in the next section.

An even stronger test is that heterotic string theory compactified on a torus gains additional non-abelian gauge symmetry for particular $O(\alpha')$ values of the parameters (torus metric and flat gauge connection), in a way not predicted by the low energy Lagrangian (2). What makes this test particularly interesting is that this happens for values of the heterotic string parameters corresponding to a weakly coupled dual type I theory, so there is no obvious mechanism for BPS states to become massless and we seem to

have a contradiction. This paradox and its very interesting resolution are described in [28].

These considerations also lead to a proposal for the strong coupling limit of the $E_8 \times E_8$ heterotic string – it is M theory compactified on an interval [18]. After compactification on S^1 , the two heterotic strings are continuously connected (using T-duality), so this establishes a complete chain of dualities connecting all the theories.

4 Dirichlet branes

A Dirichlet (D) brane can be defined in type I and II superstrings. It is simply a hypersurface in space-time on which open strings are allowed to end. [6] The open strings are quantized just as in type I theory, with the difference that the end points satisfy Dirichlet boundary conditions $X^\mu(0) = X^\mu(\pi) = x^\mu$ for the coordinates normal to the hypersurface. This makes sense for a hypersurface of any number of dimensions, say p space and one time, but consistency ultimately restricts p to one of the values for which the form $C^{(p+1)}$ or its dual $C^{(7-p)}$ appears in the Lagrangians (1) or (3).

Just as quantizing open superstrings in ten dimensions leads to ten-dimensional super Yang-Mills theory, the new open string sector also contains a supersymmetric gauge theory as its massless sector. The vector $A_\mu(x)$ is reduced to a $p+1$ -component vector, while its other components are replaced by scalar fields X^i describing fluctuations of the original hypersurface. In the low energy, low amplitude limit, the Lagrangian is the $d = 10$ super-Maxwell Lagrangian (a free theory) with the substitution $A_i \rightarrow X^i/\alpha'$.

The identification of D-branes with the BPS RR charged p -branes rests on their tension, charge and the fact that they preserve half of the ten-dimensional supersymmetry [25]. The last statement is the simplest to see as it generalizes a statement we made for the type I string: the boundary conditions of an open string relate the two world-sheet spinors θ^1 and θ^2 , but the zero modes of a linear combination of the two remain. Similarly, the relation $T_p \sim 1/g_s$ generalizes the $1/g_s = e^{-\phi}$ prefactor of the type I open string action (3).

Comparing to the p -brane discussion, the fields X^i describe small variations of the embedding f of (5). The additional world-volume gauge theory is new but could have been predicted from considerations of supersymme-

try. The real novelty of the D-brane appears when several parallel branes are brought into contact. Now open strings stretching from one D-brane to another produce new states, becoming massless when the D-branes coincide.

The open strings can again be distinguished by Chan-Paton indices at each end, say $1 \leq i \leq N$, each labelling a choice of D-brane. The discussion is exactly as for type I and indeed type I open strings can be regarded as associated with 32 D9-branes filling space-time. The open string fields again become matrices, and are governed by dimensionally reduced SYM with gauge group $U(N)$. The Lagrangian is

$$\mathcal{L} = \frac{1}{g_s} \text{Tr} F^2 + \frac{1}{g_s \alpha'^2} \text{Tr} (DX)^2 + \bar{\psi} \not{D} \psi + \frac{1}{g_s \alpha'^4} \sum \text{Tr} [X^i, X^j]^2. \quad (7)$$

(For $p = 3$ this is exactly the $\mathcal{N} = 4$ SYM of section 2, and we are giving a different picture for the physics reviewed there.) Separating the branes in space corresponds to giving a vev to the matrix X^i . The moduli space of configurations with zero energy is $\mathbf{R}^{N(9-p)}/S_N$, the space of diagonal constant matrices $X_{mn}^i = x_n^i \delta_{mn}$ (solutions of $[X^i, X^j] = 0$ modulo gauge transformations). x_n is the position of the n 'th D-brane, and if all x_n are distinct, the gauge symmetry is broken to $U(1)^N$. By the standard analysis of the Higgs effect, off-diagonal states $A_{\mu\ mn}$ and X_{mn}^i will get masses

$$m^2 = \frac{1}{\alpha'^2} |x_m - x_n|^2. \quad (8)$$

In other words, an open string stretched from brane m to n gains a mass equal to the distance $|x_m - x_n|$ multiplied by the string tension $1/\alpha'$.

Perhaps the simplest imaginable physical application of this is to propose that the observable universe is a set of three-branes embedded in ten-dimensional space, and that we have just described the origin of the gauge symmetry of the Standard Model. The breaking of gauge symmetry is then associated with separating individual three-branes in the other dimensions.

A potential problem with this idea is that the gravitational interaction will not in general be described by four-dimensional general relativity, as it is a 'bulk' ten-dimensional phenomenon. This problem can be solved if the additional dimensions are a small compact space. To judge how small is small enough, one must keep all Kaluza-Klein gravitational modes in the effective four-dimensional theory and compare their effects with the experi-

mental bounds on deviations from general relativity. These bounds are weak and allow sizes on the order of a millimeter!⁶

4.1 Remarks on the relation to noncommutative geometry

The promotion of space-time coordinates to matrices certainly sounds like it deserves the name “noncommutative geometry,” and indeed the picture we just described has a noteworthy similarity to Connes’ construction of Yang-Mills theories in terms of the noncommutative geometry of a discrete bundle over space-time. [7] In both pictures, the underlying space-time is a product of the original space-time with a set of points labelled by a “fundamental index,” so points correspond to pairs (x, m) . In both pictures, gauge symmetry breaking is associated with a non-zero distance between the points (x, m) and (x, n) .

Making this correspondance more precise reveals some differences. One is that the D-branes are embedded in a higher-dimensional space-time, so the full theory must contain more degrees of freedom. A more important difference is the relation between distance and gauge symmetry breaking. In physical terms, the Higgs fields and gauge symmetry breaking of [7] are exactly what would be produced by straightforward dimensional reduction of a higher dimensional gauge theory, $\phi_i = A_i$ with all fields taken independent of the additional coordinates X . A goal of [7] is to define geometric concepts such as distance in terms of natural operators, in this case the covariant derivative D acting on functions $f_m(x)$. The distance associated to this operator is the difference $|f_m(x) - f_n(y)|$ maximized over all functions with $|Df| \leq 1$. Evaluating this for two points (x, m) and (y, n) with $x = y$ produces the distance $d(m, n) = 1/\phi_{mn}$. In other words, the Higgs has units of inverse length.

Although this agrees with the usual dimensional analysis of field theory, it is not the relation (8) of the D-brane construction. In string theory, the presence of a fundamental constant with units of $(\text{length})^2$ allows the scale of gauge symmetry breaking to be proportional to a distance $X_{mm} - X_{nn}$,

⁶ So far, similar models which have been studied in more detail (for example in [3]) have other dynamical constraints which keep this size microscopic, but we do not know a general argument requiring this.

and zero distance to correspond to enhanced gauge symmetry.

The meaning of this difference is made clearer by considering compactification on a torus, after which the dimensional reduction origin for scalars ϕ and the D-brane origin for scalars X are related by T-duality [6]. One way to say this is that in this case the D-brane open string theories become secretly ten-dimensional once all the degrees of freedom are included. The additional degrees of freedom which appear on the torus are open strings which start on D-brane m , wind any number of times w^i , and end on D-brane n . A field describing all of these modes would be $A_{mn}(x, w)$. This is Fourier dual to $A_{mn}(x, \tilde{X})$ on a dual torus and the statement of T-duality is that these degrees of freedom are local fields on the dual torus, with radius α'/R . Furthermore, displacing one of the original D-branes by varying $X_{mm}^i(x, 0)$ is T-dual to varying the background flat connection on the dual torus, $\phi_{mm}^i = \int d\tilde{X}^i A_{i,mm}(x, \tilde{X})$. Reversing the T-duality, Higgs fields ϕ_{mm}^i originating by dimensional reduction in string theory are proportional to a physical distance on the dual torus.

4.2 Physics of coincident branes

The idea that when two solitonic objects approach each other, extended objects stretched between them become light, leading to new massless degrees of freedom when they coincide, is clearly very general. Indeed, dualities can be used to relate every object in superstring theory to a Dirichlet brane (at least in some limit of moduli space), showing that this is generic.

One role of these degrees of freedom is to produce bound states of several D-branes. String duality predicts additional BPS solitons with non-unit charge, for example the dyonic strings of type IIB theory. These have been shown to exist as bound states of D-strings and fundamental strings [40].

The most striking and important application of this so far is due to Strominger and Vafa [35], and extended by many workers. They considered D-brane bound states which are continuously connected (by varying the vacuum moduli) to extremal black holes with event horizons. These are BPS, which guarantees that the number of states of a given charge is the same for the two systems. It can be calculated in the weak coupling limit of the D-brane system, and thus the Bekenstein-Hawking entropy of extremal black holes can be calculated from first principles in string theory. This direction is advancing rapidly and any list of references would be out of date by the

time this reaches print.

Although the matrix nature of the D-brane coordinates X^i is important in describing the full dynamics, the moduli space of D-branes in flat space was the subspace $[X^i, X^j] = 0$. At low energies where the moduli space approximation is good, non-commutativity plays no role, and the embedding into the full non-commutative configuration space is somewhat trivial.

However there are other geometries in which the non-commutativity plays a more essential role, and the moduli space is embedded non-trivially in the full configuration space. An example is the description of D-branes in an ALE space found in [12]. ALE spaces are four real dimensional spaces with self-dual metrics asymptotic to the flat metric on \mathbf{R}^4/Γ , with Γ a discrete subgroup of $SU(2)$. Self-dual metrics are hyperkähler and these metrics were constructed by Kronheimer as hyperkähler quotients [23]. Now the moduli spaces of supersymmetric gauge theories (with 8 supercharges, as in the present case) are (by definition) hyperkähler quotients [17, 16], and so an ALE space can be obtained as the moduli space of a known SYM Lagrangian. In ref. [12] it was shown that, starting with \mathbf{R}^4 and an action of Γ , the natural definition of D-branes on the quotient is just this Lagrangian. One introduces ‘image’ D-branes on whose Chan-Paton index the regular representation of Γ acts, and takes the subsector of (7) invariant under the simultaneous action of Γ on \mathbf{R}^4 and this index. The connection with the previous examples is that the construction relies on massless degrees of freedom which appear when a D-brane coincides with its images.

More explicitly, we let $g \in \Gamma$ act on the fields of (7) with $N = |G|$ as $A_\mu \rightarrow r(g)A_\mu r(g)^{-1}$ and $X^i \rightarrow R_j^i(g)r(g)X^j r(g)^{-1}$. $r(g)$ is the regular representation of Γ and $R_j^i(g)$ a unitary representation in $SU(2) \subset SO(4)$. The Γ -invariant subsector has gauge group the unitary group of $L^2(\Gamma)$, and the moduli space of gauge equivalence classes of vacua is \mathbf{C}^2/Γ .

The string theory contains additional fields ζ^a which naturally couple to the Lagrangian (7), and deform the relation $[X^i, X^j] = 0$ to

$$[X^i, X^j] = \zeta^a \sigma_a^{ij} \quad (9)$$

(where σ_a^{ij} are Pauli matrices generating the Lie algebra of the commuting $SU(2)' \subset SO(4)$). The $[X^i, X^j]$ are exactly the moment maps of the hyperkähler quotient construction, and after this deformation the moduli space is smooth with an ALE metric.

Thus the position degrees of freedom of D-branes on a manifold of non-trivial topology and geometry are embedded as non-commuting variables in a larger configuration space of open string degrees of freedom. We will return to the physics of this in the next section.

4.3 Non-identical branes

So far we discussed parallel identical D-branes, but one could take D-branes of different p , or non-parallel D-branes, and stretched strings between them with mass $m^2 = m_0^2 + |\Delta X|^2$ appear in every case. The physics depends on m_0^2 , the minimal (mass)² for a bosonic string (fermionic strings always have $m_0 = 0$). Generically $m_0^2 < 0$ and at small ΔX the theory becomes unstable. The physics of this is quite interesting – for example the annihilation of a p -brane with an anti- p -brane begins with this instability [5] – but still rather mysterious.

Certain geometries preserve some supersymmetry, which guarantees that $m_0^2 \geq 0$. One example is the D1-brane of type I theory, which implicitly is contained in its D9-branes. This is the heterotic string soliton required by type I–heterotic duality. Its additional degrees of freedom are strings stretching from 1-brane to 9-brane; they will have a single $SO(32)$ Chan-Paton index, so they must be the origin of the heterotic string fermions λ^I of section 3. Indeed one can check that $m_0^2 > 0$ for bosons and the only new massless states are these chiral fermions.

The D5-brane of type I theory also has strings stretched to the 9-branes; now $m_0^2 = 0$ so there are additional massless charged scalars and a non-trivial moduli space. Our earlier identification of the five-brane with a self-dual solution of the Yang-Mills equations leads to an interesting result – this moduli space must be isomorphic to the moduli space of instantons. It turns out that this description is exactly the ADHM construction of moduli space [41]. Furthermore, the action of the D1-brane heterotic string soliton in the presence of the five-brane should be equivalent to the sigma model action for strings coupled to the self-dual solution, and making this explicit reproduces the ADHM construction of self-dual gauge connections [10].

5 Short Distances in Superstring theory

A long-standing belief of string theorists was that the finite size of the string determined a minimum distance, below which the standard concepts of space-time and metric would break down. Many precise forms of this statement were made.

One argument for this observes that gravitons, fluctuations of the metric, are particular modes of the string. At scales shorter than the string scale, there is no preferred way to separate these modes from the other modes of the string and define a unique space-time metric. This is a real problem with the idea that metric could be an appropriate description of small-scale geometry, and we will return to it below.

In studying structure at small scales, one should try to distinguish properties of the background space-time from properties of the object one uses to probe it. Although in quantum gravity, any probe will affect the background, clearly we should try to use the smallest and lightest probe we can find. However, the mass (or tension) of the probe controls its own quantum fluctuations (it plays the role of $1/\hbar$ in (5)) and this leads to an uncertainty relation: in quantum gravity, the minimal length scale which can be resolved is the Planck length [8]. In superstring theory, such reasoning leads to the conjecture that the minimal length scale should be the ten-dimensional Planck length as defined in the Einstein metric, $1/l_{p10}^8 = 1/\alpha'^4 g_s^2$. To resolve it, we need a probe smaller than the fundamental string.

Dirichlet branes have a sort of intermediate status between fundamental states and conventional solitons, because of their anomalously small charge and tension $T \sim 1/g_s$. Since Newton's constant is $1/G_N \sim 1/g_s^2$, the gravitational (and other) fields around a Dirichlet p -brane behave as $g_{ij} \sim g_s/r^{7-p}$. As stressed by Shenker [33], in the weak coupling limit, the size of the brane, identified as the size of the region in which the fields become strong, shrinks to zero as $l_p \sim g_s^{1/(7-p)} \sqrt{\alpha'}$. For the natural probes ($p = 0$ in IIa or $p = 1$ in IIb and type I), this is larger than l_{p10} but smaller than $\sqrt{\alpha'}$.

However, because the derivatives of the fields become large in this region, using the low energy Lagrangian to make this argument is not valid. To correct the argument, we must consider the details of how these fields and interactions are realized in string theory.

When we bring two branes near each other, they will interact. In the original field theory, the two branes are described by a complicated multisoliton

solution. In the limit of large separation, the interaction is mediated by the long-range massless fields of (1) or (3). Their leading long-range behavior depends only on the mass and charge of the source, and a good approximation for the action of the combined system is obtained by substituting this into the p -brane action (5) of the other brane.

The D-brane description of the interaction is very different. New open string degrees of freedom appear. Even if we do not excite them directly, in the quantum theory they will have zero-point energies, which can depend on the distance between the branes and the other parameters of the system. This produces a sort of ‘Casimir effect’ interaction between the branes.

Although these two descriptions of the interaction are quite different, the same diagrams in perturbative string theory are responsible for both of them. This is simplest to see in the D-brane description and at leading order in the string coupling. This contribution is mediated by a string world-sheet with annulus topology, and with its two boundaries constrained to sit on one or the other D-brane.

The diagram can be interpreted in field theory terms in two ways. We can regard it as a sum of closed strings emitted by the first D-brane, propagating and absorbed by the other. This includes the classical gravitational and abelian gauge interactions, along with the exchange of an infinite tower of massive modes of the closed string. Alternatively, we can regard it as a sum of loop contributions from open strings stretched between the branes, the lightest having $(\text{mass})^2 = m_0^2 + (\Delta X)^2$, but again including an infinite tower of stringy excitations.

World-sheet duality states that we do not add these two sums but rather that they are two descriptions of the same physical amplitude. However, the closed string sum is a more physical description when the separation between the branes is much larger than the string scale $\sqrt{\alpha'}$, while the open string sum is more physical if the separation is much smaller. This is because the leading behavior in either of the limits is entirely produced by the leading term in the appropriate sum, the contribution of the lightest states. To a very good approximation, the infinite towers of more massive states can be neglected, returning us to a field theory description. But the relevant field theory is different in the two regimes.

Thus string theory incorporates both pictures of the interaction between branes, and provides an interpolation between them. Having seen this, we now know that at short distances, we should not trust even the qualitative

behavior of the original supergravity solutions describing the branes. The correct description of the short distance interaction between branes is given by the supersymmetric gauge theory (7) and its generalizations.

5.1 D0-brane scattering

A good example which illustrates the resulting physics is the scattering of two D0-branes. This was studied semiclassically in [2] and in terms of open string quantum mechanics in [9, 22]. It was shown in [11] that each of these approximations is controlled in a specific physical regime, leading to definite results for energies below the D0-brane mass, and thus probing length scales $l > g_s \sqrt{\alpha'}$.

Gravitational scattering of point particles is always ‘hard’ with a significant amplitude for large deflection, just like the Rutherford scattering of electrons off nuclei, and for the same reason – the interaction grows at short distance like an inverse power of r . However, D0-branes scatter differently, because the gravitational interactions are not fundamental but are derived from the quantum fluctuations of open strings. For any fixed total energy E and corresponding velocity v (with $E = v^2/2g_s$), there exists a distance $b \sim \sqrt{v}$ such that at $r < b$ the open string effects cannot track the motion of the D-branes (the quantum mechanical Born-Oppenheimer approximation breaks down) and the force law is modified. This leads to a softer scattering, controlled (surprisingly from the ten-dimensional string theory point of view) by the momentum measured in units of the eleven-dimensional Planck length. Indeed the importance of this scale in D0-brane physics was predicted by a simple scaling argument in [22].

One can show [11] that in this problem the quantum open string effects also reproduce the long range fields; in particular the motion of the D-branes at long distances is described geometrically, as one D-brane moving in the metric produced by the other. The fact that the gauge theory description could reproduce this exactly is special to this and a few other problems with extended supersymmetry, but in general there must be a geometrical description of the motion interpolating between the known long-distance metric and a short-distance metric computable from the D-brane gauge theory – essentially, the natural metric on the moduli space of ground states of the gauge theory. This identification of the short-distance metric as a derived quantity in the D-brane gauge theory escapes the paradox mentioned at the start of

the section, that the original graviton mode of the string field has no sensible definition at sub-stringy scales.

An important lesson of the scattering exercise is that the geometrical description breaks down at very short distances. The lower the energy of our probes, the shorter the distance scale of the breakdown, but such a breakdown will always occur whenever the original geometry was singular. This is a special case of a general principle in field theory: singularities in effective field theory are produced by integrating out states which become massless, i.e. keeping only the functional dependence of their quantum effects on the other degrees of freedom. By keeping their full dynamics, one obtains a non-singular description. The effective field theory here governs the motion of the D-branes at low velocity; the states we integrate out are the open strings stretched between the D-branes; they become massless when the D-branes coincide, and by keeping them in and treating the full theory (7) we get the true, non-singular dynamics.

To illustrate how these concepts apply in a situation with non-trivial topology, we reconsider the description of ALE space given in section 4.2. The remarks above motivate the claim that this is the appropriate description whenever the volumes of the homologically non-trivial two-cycles in the space are much less than the string scale, $\text{Vol}(S^2) \ll \alpha'$, while the normal metric and manifold description is appropriate when $\text{Vol}(S^2) \gg \alpha'$. The continuous connection between the two descriptions this implies is still somewhat mysterious, though the interpolation should be smooth.

For definiteness we consider a single D0-brane. At low energies its motion will be governed by its gauge theory moduli space metric, which is just the ALE space. However this is the submanifold defined by (9) of a larger space, topologically trivial but carrying a natural noncommutative algebraic structure. At finite energies the dynamics can explore more of this configuration space, and when the energy exceeds the height of a saddle point of the gauge theory potential, $E \sim (\text{Vol}(S^2)/\alpha')^2/g_s$, the topology of the available configuration space changes. In this regime the original space-time interpretation is completely inadequate – it sits in something more fundamental. This will be true even at low energies if $\text{Vol}(S^2) \sim g_s^{2/3}\alpha' = l_{p11}^2$, as quantum fluctuations will dominate the effects of the potential. D0-brane probes indeed see a minimum length scale, but it is the eleven-dimensional Planck length.

To summarize, space-time indeed has a geometric description at sub-

stringy scales. This is space-time as seen by D-branes, but these are the only objects we know in string theory which could resolve such scales. The metric is no longer fundamental but is instead derived from other fundamental degrees of freedom, the open strings responsible for the dynamics of the D-brane. Since ten-dimensional quantum gravity is not valid at short distances, the ten-dimensional Planck length plays no role. The eleven-dimensional Planck length plays a surprisingly important role, perhaps pointing to a more fundamental role for M theory.

The geometric picture of space, either around a brane or in non-trivial topology, is an approximate description valid at low energies. When D-branes approach within short distances or encounter small scale structures, new degrees of freedom become light, and the correct description of the dynamics is to treat the new degrees of freedom on an equal footing with the original coordinates. Thus the original geometric description is naturally embedded in a larger configuration space, which in these examples is a matrix generalization of the algebra of space-time coordinates.

6 Further Directions

In this talk we only described ten and eleven-dimensional theories. Much of the beauty and interest of superstring duality appears after compactification to lower dimensions. This is a vast subject and we will confine ourselves to the single remark that the results motivate the point of view that all of the p and D-branes we have discussed are in some deep sense the same [37]. The evidence is that for every soliton, there is some limit in which it plays a fundamental role. For example, a p -brane gives rise to a string after compactification on T^{p-1} , by wrapping it around the T^{p-1} (i.e. taking its hyperplane to be $T^{p-1} \times \mathbf{R}^2$). The lower dimensional theory then has a discrete duality symmetry which exchanges this object with the fundamental string.

Much remains to be understood about short distance structure in string theory. One would conjecture that every local structure with non-trivial topology has a specific small scale realization as an embedding in a larger configuration space. How this description matches on to normal geometry and topology and the role of general covariance in the complete picture are quite mysterious. It is unclear whether supersymmetry plays an essential

or merely facilitating role in the discussion. The treatment of [11] allows studying shorter lengths than l_{p11} but is valid only up to energy scales $E \sim 1/g_s$ at which pair production of D-branes becomes possible; it is unclear whether space-time is sensible at smaller scales or what should replace it.

Eleven-dimensional M theory is at the very least a new regime of the single unified theory with quite different properties than string theory, and very possibly more fundamental. M theory contains no fundamental string but instead a membrane and five-brane. The IIA string arises as the membrane wrapped around the S^1 . One naturally wonders if the membrane can be quantized and used as the fundamental object. It should be realized that unlike quantizing the superstring, this is not a problem of quantizing a known classical Lagrangian. The membrane world-volume theory is non-renormalizable and its physics depends strongly on the specific cutoff used to define it. This is in contrast to the string whose renormalizability implies that independent of their short-distance definition, all superstrings will have the same long distance physics as the fundamental string.

Direct attempts at membrane quantization have not yet succeeded. Perhaps the D0-branes of IIA theory are closer to the fundamental M theory degrees of freedom, as in the work [4] (which also finds interesting connections with noncommutative geometry).

6.1 A complete formulation of string/M/F/...-theory

This talk was addressed to both physicists and mathematicians, and perhaps for the mathematicians, there is another question which would seem to have priority over the two I listed in the introduction: what are we talking about? In other words, can we define the theory. Given that this is hard even for field theory (as we mentioned in section 2, giving a definition of $\mathcal{N} = 4$, $d = 4$ SYM for which duality can be precisely formulated would be a real advance) I will only make some general comments.

Let me first comment that there is striking evidence that we have all the degrees of freedom. The Bekenstein-Hawking entropy of a black hole comes out of very general principles of general relativity, quantum field theory and thermodynamics, with no dynamical input. Strominger and Vafa's success in reproducing this from microscopic dynamics, and subsequent success in extending this to non-BPS black holes, strongly suggests that there are no hidden degrees of freedom waiting to be discovered. If so, we have some solid

ground from which to start.

In the light of duality, we should start by re-examining just what we mean by a complete definition. Generally, we start off with a list of physical properties we believe our theory possesses – a consistent probability interpretation (for quantum theories), locality and causality, and a symmetry group. Usually a definition will not make all of these properties manifest and we must prove the others.

Usually quantum field theory is based on a set of local fundamental degrees of freedom, which become independent in some limit. But, in a theory with duality, it is not natural to single out a particular set of degrees of freedom as fundamental. When we do, the physics of the other limits becomes inaccessible.

One might try to make a precise definition for each possible choice of fundamental object, show that these have overlapping regimes of validity, and find some sort of explicit change of variables or ‘transition function’ on the overlaps [38]. This is of course just what we have been doing, but the variables have not been defined precisely, and the change of variables is only known explicitly for an abelian subsector of the theory, non-interacting at low energies.

The ideal definition of string/M/F/...-theory would have a sort of ‘manifest duality’ in which every object which could become fundamental in any limit was included as a fundamental degree of freedom. However, there is a bewildering variety of candidate fundamental degrees of freedom, especially after compactification, and such a description might well require intractable constraints among them.

One might look for some general ‘principle of construction’ that builds up all possible extended objects as composite objects. Perhaps the appropriate constituent degrees of freedom have not even made their appearance yet!

The assumption of local fundamental degrees of freedom is not logically necessary, and more general frameworks have been studied, only imposing locality on the observables. Indeed locality has never been well understood even in perturbative string theory. For example, there exists no formulation of the state at a specified time in string theory, so the initial value problem cannot even be properly stated, much less shown to be causal.

These are all indications that a fully satisfactory formulation of superstring theory or M theory is still far beyond us. At least we now have some feeling for the non-perturbative physics it is supposed to represent. New in-

sights from both physicists and mathematicians will surely be required before we have it.

I would like to thank the organizers and participants for a stimulating school, the Rutgers high energy theory group and its many 1995-96 visitors for teaching me about many of these topics and for enjoyable collaboration, and Scott Axelrod, Alain Connes and Is Singer for giving me some idea of a mathematician's point of view on these topics.

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